How to make topological superconducting wires?

Prerequisites :

(1) Single species of fermion [single spin] $\Rightarrow$ SO + B-field

(2) Superconductivity

Last time semi-conducting wire with

(1) tunable μ

(2) spin orbit

(3) B-field

(4) proximity superconductivity

$$H = \psi_p^\dagger \left(\left[\frac{p^2}{2m} - \mu \right] \tau^z + \alpha p \sigma^z \tau^z + B \sigma^x + \Delta \tau^x \right) \psi_p$$

where $\psi_p^\dagger = \left(\psi_{\uparrow p}^\dagger \; \psi_{\downarrow -p}^\dagger \; \psi_{\downarrow -p} \; -\psi_{\uparrow p} \right)$

"Band structure"

Let's study the "tight" binding version of this Hamiltonian

$$\sum_p \frac{p^2}{2m} c_{p\sigma}^\dagger c_{p\sigma} + \alpha p \sigma^z c_{p\sigma}^\dagger c_{p\sigma} \rightarrow \sum_i \Big[\underbrace{-t\, e^{-i\chi\sigma}}_{-2t\cos(p \pm \chi)} c_{i\sigma}^\dagger c_{i+1\sigma} + h.c. \Big]$$

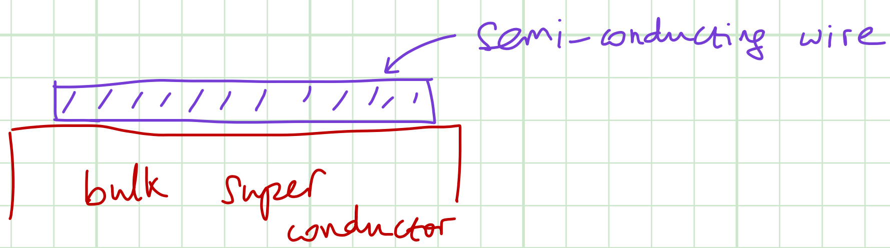
sign depends on spin

Let's plot the band-structure with Mathematica

$\Rightarrow$ Expectations?

Open ended wire

consider a segment of a (non)topological SC. wire

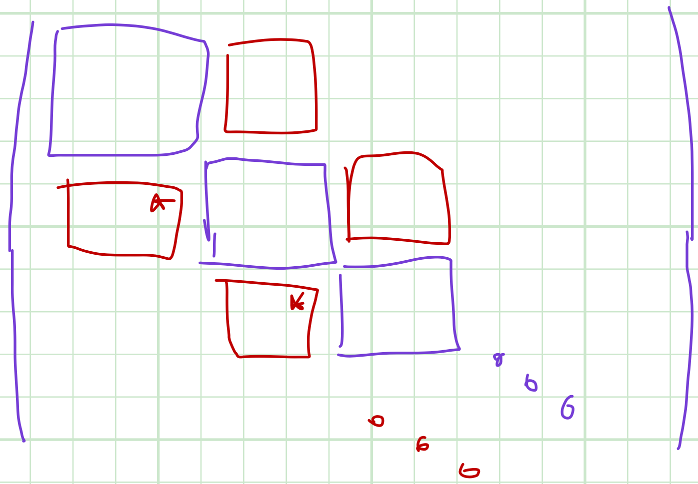


The real space wave function is

$$\psi^\dagger = \left(c^\dagger_{1\uparrow}\, c^\dagger_{1\downarrow}\, c_{1\uparrow}\, c_{1\downarrow}\, c^\dagger_{2\uparrow}\, c^\dagger_{2\downarrow}\, c_{2\downarrow}\, c_{2\uparrow} \dots \right)$$

[Note: I had to double the # of fermions and, hence, modes]

$\psi^\dagger H \psi =$



$$\square = \left(\begin{array}{cc|cc} 2-\mu & B & & \Delta \\ B & 2-\mu & -\Delta & \\ \hline & -\Delta^* & \mu-2 & -B \\ \Delta^* & & -B & \mu-2 \end{array}\right)$$

$$\square = \begin{pmatrix} e^{ix} & & & \\ & e^{-ix} & & \\ & & -e^{-ix} & \\ & & & -e^{ix} \end{pmatrix}$$

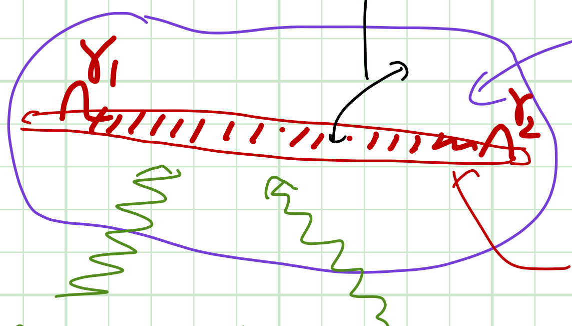
$\Rightarrow$ show appearence of zero mode

$\Rightarrow$ Decay of zero mode with distance

Topological superconductivity in ultracold atoms (proposal only)

RF coupling to enable tunneling of pairs between wire and BEC

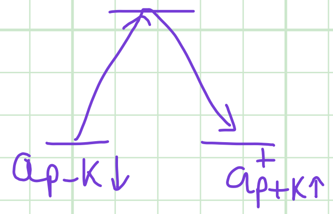
BEC cloud (proximity pairing)



1D trap $\Rightarrow$ Kitaev wire

Raman Beams to induce spin-orbit and B-field.

Spin orbit and B-field using Raman coupling



where $2k$ = momentum transfer from the two Raman photons.

$$H_{Raman} = \left(a^{+}_{p-k\downarrow} \quad a^{+}_{p+k\uparrow}\right)\begin{pmatrix} 0 & \overset{=B}{\frac{\Omega_1\Omega_2}{\delta_e}} \\ \frac{\Omega_1\Omega_2}{\delta_e} & 0 \end{pmatrix}\begin{pmatrix} a_{p-k\downarrow} \\ a_{p+k\uparrow} \end{pmatrix}$$

$$H_{KE} = \sum_{k\sigma} a^{+}_{p\sigma}\left(\frac{p^2}{2m} - \mu\right) a_{p\sigma}$$

Apply a spin-dependent boost:

$$\left.\begin{aligned} p-k,\downarrow &\rightarrow p\downarrow \\ p+k,\uparrow &\rightarrow p\uparrow \end{aligned}\right\}$$

$$B\, a^{+}_{p-k\downarrow}\, a_{p+k\uparrow} \rightarrow B\, a^{+}_{p\uparrow} a_{p\downarrow} \qquad a^{+}_{p\uparrow}a_{p\uparrow}\left(\frac{p^2}{2m} - \mu\right) \rightarrow a^{+}_{p\uparrow}\, a^{+}_{p\downarrow}\left(\frac{(p+k)^2}{2m} - \mu\right)$$

$$B\, a^{+}_{p+k\uparrow}\, a_{p-k\downarrow} \rightarrow B\, a^{+}_{p\downarrow}\, a_{p\uparrow} \qquad a^{+}_{p\downarrow}a_{p\downarrow}\left(\frac{p^2}{2m} - \mu\right) \rightarrow a^{+}_{p\downarrow}\, a_{p\downarrow}\left(\frac{(p-k)^2}{2m} - \mu\right)$$

$$H_{Raman} + H_{KE} \rightarrow \sum_{p} \left(a^{+}_{p\uparrow} \quad a^{+}_{p\downarrow}\right)\left(\frac{p^2}{2m} + \frac{kp}{m}\sigma^z - \mu + \frac{k^2}{2m} + \sigma^x B\right)\begin{pmatrix} a_{p\uparrow} \\ a_{p\downarrow} \end{pmatrix}$$

Topological Superconductivity in semiconducting nanowires

Lutchyn, Sau, Sarma PRL <u>105</u> 077001 (2010).
Oreg, Refael, von Oppen PRL <u>105</u> 177002 (2010). } theory
Mourik, ..., Frolov, ..., Kouwenhoven Science <u>336</u> 1003 (2012). } experiment

Topological superconductors in ultracold atoms

Jiang et al PRL <u>106</u> 220402 (2011). ⇐ theory

Braiding Majorana fermions

See Ivanov PRL 86, 268 (2001).

Exchange of 2 Majorana fermions

$$\gamma_1 \to \gamma_2$$
$$\gamma_2 \to -\gamma_1$$ ← This sign has to do with crossing a branch cut

$$\Rightarrow U(\sigma_i) = \exp\left(\pi/4\ \gamma_{i+1}\gamma_i\right) = \frac{1}{\sqrt{2}}\left(1+\gamma_{i+1}\gamma_i\right) \equiv \exp\left[i\frac{\pi}{4}\left(2f_i^+f_i - 1\right)\right] = \exp\left[i\frac{\pi}{4}\sigma^z\right]$$

$\circ$ $\circ$ $\circ$ $\circ$

γ_1 γ_2 γ_3 γ_4 use f_1, f_3 basis

$$\gamma_1 \curvearrowright \gamma_2\ \gamma_3\gamma_4 \Rightarrow e^{i\frac{\pi}{4}\sigma_1^z} \otimes I = \begin{pmatrix} e^{-i\pi/4} & & & \\ & e^{i\pi/4} & & \\ & & e^{-i\pi/4} & \\ & & & e^{i\pi/4} \end{pmatrix} \begin{array}{l} |00\rangle = |vac\rangle \\ |10\rangle = f_1^+|vac\rangle \\ |01\rangle = f_3^+|vac\rangle \\ |11\rangle = f_1^+f_3^+|vac\rangle \end{array}$$

$$\gamma_1\gamma_2\ \gamma_3 \curvearrowright \gamma_4 \Rightarrow I \otimes e^{i\frac{\pi}{4}\sigma_3^z} = \begin{pmatrix} e^{-i\pi/4} & & & \\ & e^{-i\pi/4} & & \\ & & e^{+i\pi/4} & \\ & & & e^{+i\pi/4} \end{pmatrix}$$

$$\gamma_1\ \gamma_2 \curvearrowright \gamma_3\ \gamma_4 \Rightarrow \frac{1}{\sqrt{2}}\left(1+\gamma_3\gamma_2\right) = \frac{1}{\sqrt{2}}\left(1+i\left(f_3^+ + f_3\right)\left(f_1^+ - f_1\right)\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & -i \\ 0 & 1 & -i & 0 \\ 0 & -i & 1 & 0 \\ -i & 0 & 0 & 1 \end{pmatrix}$$

$$= \underbrace{\frac{1}{\sqrt{2}}}_{\mathbb{I}} + \frac{1}{\sqrt{2}}\, i\Big[\underbrace{f_3^+f_1^+}_{-|11\rangle\langle 00|} + \underbrace{f_3f_1^+}_{-|10\rangle\langle 01|} - \underbrace{f_3^+f_1}_{|01\rangle\langle 10|} - \underbrace{f_3f_1}_{|00\rangle\langle 11|}\Big]$$

$$f_3^+f_1 \Rightarrow \langle vac|f_3f_1\ f_3^+f_1^+|vac\rangle = -\langle vac|f_3f_1f_1^+f_3^+|vac\rangle = -1 \Rightarrow -|11\rangle\langle 00|$$
$$f_3f_1^+ \Rightarrow \langle vac|f_1\ f_3f_1^+f_3^+|vac\rangle = -\langle vac|f_1f_1^+f_3f_3^+|vac\rangle = -1 \Rightarrow -|10\rangle\langle 01|$$
$$f_3^+f_1 \Rightarrow \langle vac|f_3\ f_3^+f_1\ f_1^+|vac\rangle = 1 \Rightarrow |01\rangle\langle 10|$$
$$f_3f_1 \Rightarrow \langle vac|f_3f_1\ f_1^+f_3^+|vac\rangle = 1 \Rightarrow |00\rangle\langle 11|$$

$\Rightarrow$ This set of generators defines the Braid group for the 4-Majorana zero mode chain.

Universal quantum computer can be implemented using e.g.

$R(\pi/8)$ + Hadamard + CNOT (Wikipedia)

$\Rightarrow$ except for the $R(\pi/8)$ gate these can be implemented using braiding

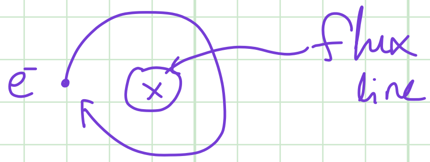
(but do require ancilla qubit(s))

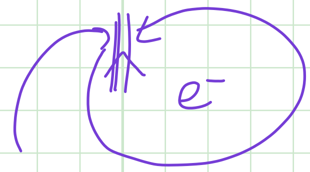
e.g.: CNOT gate can be obtained from 6 Majorana zero mode chain

by Braiding $\Rightarrow$ see Hyart ... Beenakker arXiv: 1303.4379

The 4-π Josephson effect

Aharonov-Bohm effect $\Longleftrightarrow$ Aharonov-Casher effect

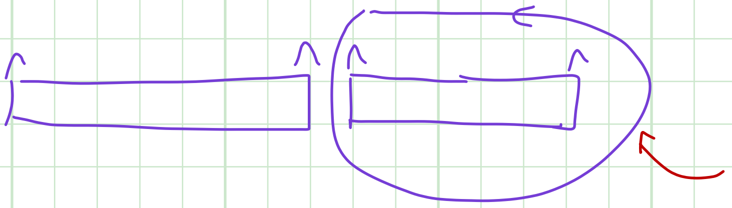




flux around charge

$\Rightarrow$ vortex around charge

$\Rightarrow$ "$-$"



phase slip $\simeq$ vortex

$\Rightarrow$ change sign of enc. fermion

$\Rightarrow$ parity of middle.